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Contextual Behaviour in a Complex World: from Voting Preferences to Financial Expectations

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Abstract

Recent years were characterized by an increasing of interest for applications of the mathematical formalism of quantum theory and its methodology to information processing and human choice behaviour in cognition, decision making, artificial intelligence, economics and finance and also social and political science. Contextual expectations and choices in real economic and financial decision making settings, non-neutral attitudes to ambiguity and problems of complete non-knowledge are posing a challenge to standard decision theories that utilize the calculus of classical probability theory. A new area that showed a potential to cope with the non-classical decision making statistics of humans is known as quantum-like modeling. Its basic tool is quantum calculus of probabilities, which is based on operation with complex probability amplitudes and the usage of Born's rule to convert complex probability amplitudes into objective probabilities. In this paper we expose a brief review introducing the core axiomatic differences between classical and quantum probability as well as discuss the decision-making settings in which quantum probability can capture agents' non-classical beliefs, superposition of ambiguous economic and financial events, and other instances of non-classical information processing.

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1 INTRODUCTION

Quantum probability models (also alluded to as "quantum-like models") are providing an operational representation of the process of agents' decision making. The models are built on the same premises as standard decision theories under uncertainty and risk - namely the aim is to capture and predict human expectations and choice formation. There is also an endeavour to develop the necessary conditions, to endow these models with a normative appeal that would go beyond their descriptive status. Quantum probability (QP) frameworks also indicate that a different form of 'rationality' can exist that goes beyond a pure maximization of expected utility by probabilistically sophisticated agents as shown in recent studies by [4], [7] and an opinion paper by [42].

We recap that QP theory is operationally established on a representation of states and observables in a complex Hilbert space. In quantum physics one deals with contextual states of electrons and photons such as spin, and the position-momentum relationship.[a] It is important to stress that mathematical models derived from quantum physics are not "ad-hoc" modifications of classical probability theory, but are derived from its own set of axioms that allow for a precise description of the phenomena under observation, see a throughout introduction and motivation regarding the applications to human cognitive states in monographs [10], [29] and [20].

In such a modeling, a human brain is considered as a "black box". One does possess knowledge about the neuropsychological processes underlying the information processing, but captures the statistical distribution of outcomes (that can be considered as an analogue of classical subjective probabilities).

QP probability theory that is grounded in quantum (non-Boolean) logic is a different type of probability calculus. It is resting upon squaring of norms of probability waves to obtain a statistical distribution of their outcomes or to capture a representative agent's subjective expectations about their distribution. We emphasize that QP can also serve as a suitable tool to accommodate subjective beliefs of decision makers about the realizations of some random variables, considered in QP based models by [4], [7], [10], [20] and [21].

Models based on QP calculus can also well explain agents' indeterminacy in respect to outcomes of some economic or financial variables. This type of indeterminacy is depicted through superposition states that also allow to capture contextual expectations that cannot be evaluated jointly by the decision maker, see e.g., [28], [9], [29]. Interference of the complex probability amplitudes can provide an accurate measure of agents' deviations in expectations in respect to the classical probability (CP) measures. Contributions by [4] and [2] are capturing well the ambiguity aversion of decision makers, when confronted with Ellsberg-type of subjective uncertainty. Other contributions model the impact

[a] Interested reader is referred to some foundational texts on quantum mechanics ([23]) and quantum information, [40].

on asset prices and the resulting emergence of financial bubbles from agents' ambiguous beliefs that can lead to an overestimation of future returns, cf. [31] and [35]. Finally, we can also transpose the irrationality and contextuality in preferences to political choices that are often characterized by a complexity of the final outcomes. We discuss how interdependence of preferences can be captured in QP based models with the aid of *entangled states*, cf. [33] and [34].

The core aim of this paper is to show that contextuality is an omnipresent phenomenon in economic, financial and political decision making settings. Presence of subjective uncertainty (coined "ambiguity" in economic literature) can trigger a non-additivity of beliefs and a departure from Bayesian updating scheme. The same challenge pertains in respect to agents' choices that are often affected by the actual realization of the economic, financial and political states. There is a popular explanation in behavioural economics and finance that aims to focus on qualitative factors, namely, the causes or reasons behind the observed deviations from normative decision theories, [26], [44], [48] and recently, [46]. We can witness few biases and non-classical baseline beliefs can be well captured in the recognised "Prospect Theory" due to [25]. Nevertheless, the ultimate endeavour of behavioural decision theories can be considered as going beyond the depiction of some specific biases (e.g., loss aversion), to accommodate within a single framework the whole array of contextual expectations and preferences. In this setting, QP-models can serve as a suitable tool to capture and to quantify internal and external causes of non-classical beliefs and contextual preferences.

The remainder of this paper is organized as follows: in the next section (2), we present the fey features of classical and quantum probabilistic frameworks. In (3), we proceed with an analysis of the omnipresent nature of human contextuality, with some examples from non-consequential reasoning in economics and finance. We also discuss the important distinction between risk and uncertainty in financial markets, and show how agents' behaviour can bring deviations from equilibrium prices of the risky assets, (6). In (7) we illustrate with an example on US voting statistics that non-classical expectations are common in politics and voting theory. QP models can capture strong interrelation of political expectations via the notion of *entanglement*. We finalize our evaluation of the applicability to QP to contextual behaviour in the section (8), by enquiring, whether QP can be applied as a universal tool to accommodate deviations from CP. We provide a brief review of tests (based on different modifications of the seminal Bell's inequality) that aim to ascertain the violation of the bounds imposed by CP and a satisfaction of the bounds imposed by QP.

2 CLASSICAL VS. QUANTUM PROBABILITY: SOME KEY CONCEPTS

Classical probability theory (CP) was mathematically formalized by Kolmogorov (1933), [37]. CP is resting upon the calculus of probability measures, where non-negative weight $p(A)$ is assigned to any event, A.

The core property of CP that plays an important role in decision theoretic models is additivity of probabilities. For two disjoint events $A, A-$, the probability of their disjunctions equals to the sum of their probabilities:

$$P(A \cup A-) = P(A) \cup P(A-).$$

On the other side, quantum probability (QP) is a calculus of complex probability amplitudes. Instead of mathematical operations on probability measures, one uses vectors given through complex coordinates.

In quantum probability calculus, events are modeled as complex vectors and random variables are given as quantum observables.

Quantum states (wave functions) are represented by normalized vectors ψ (i.e., $\|\psi\|^2 = \langle\psi, \psi\rangle = 1$) in a complex Hilbert space $\mathcal{H}$.

The classical probability value can be realized by applying the Born's rule onto the complex amplitude (vector) ψ: Classical probability is obtained as the square of the absolute value of the complex amplitude.

$$p = |\psi|^2.$$

Complex coordinates do not need to obey additivity condition, due to the presence of interference effects between the complex probability amplitudes.

2.1 Interference as a Measure of Non-additivity

Representation of events' realization via complex probability amplitudes, instead of a direct application of probabilities, is relaxing some of the core axioms of CP. Relaxation of the distributivity axiom implies additivity of disjoint events and probability of a disjunction of two events can exhibit sub-or super-additivity.[b]

$$P(A \cup A-) < P(A) + P(A-)$$

or

$$P(A \cup A-) > P(A) + P(A-),$$

QP calculus leads to a more general formulation, aimed at quantifying the non-additivity with an additional *interference term*, where the parameter θ is determining the size of departure from non-additivity.

$$\begin{aligned} P(A \cup A-) &= P(A) + P(A-) \\ &+2\cos\theta\sqrt{P(A)P(A-)}. \end{aligned} \tag{1}$$

We recall that interference is a core feature of any waves. In the context of QP we deal with *probability waves*. When $cos\theta = 0$ there is no interference and we observe a special case in QP, whereby the additivity condition on classical probabilities is satisfied.

2.2 Non-Bayesian Update of Probabilistic Expectations

Bayesian probability update and the use of formula of conditional probability

[b] More formally distributivity would imply for events $B, A, A-$ that $p(B) \cap p(A \cup A-) = p(B \cap A) \cup p(B \cap A-)$.

lies at the heart of economic modeling of agents' expectations. Agents are presumed to update upcoming information (subjective and objective probabilities) in a Bayesian manner. In order to update their expectations after observing some informational signals, agents use Bayes formula, defined [37].

Bayes formula in the definition of conditional probability has the form:

$$P(B|A) = \frac{P(B \cap A)}{P(A)}, \ P(A) > 0. \quad (2)$$

A prerequisite of Bayesian update is the satisfaction of commutativity axiom, whereby $P(A \cap B) = P(B \cap A)$. Conditional probability also serves as a core component in the formula of total probability that gives the marginal probability of an event, conditioned on the different paths of its realization (events, states of the world). Formula of total probability (henceforth FTP) together with Bayesian networks is also serving as an important computational tool in information retrieval and artificial intelligence structures, cf., [24].

More generally, for a pair of discrete random variables, A and B, the formula can be expressed as:

$$P(B=\beta) = \sum_{\alpha} P(A=\alpha)P(B=\beta|A=\alpha). \quad (3)$$

Thus, the probability distribution of the realization of the random variable B can be computed from the probability distribution of A and the conditional probabilities $P(B=\beta|A=\alpha)$.

Given the wide-ranging experimental evidence on non-satisfaction of the Bayesian inference by humans (due to violation of one, or several axioms of CP) QP can potentially serve as a remedy, due to its different, more general calculus that allows for incompatibility of events and hence, for the non-satisfaction of commutativity and distributivity rules. Superposition state representation has a close analogue with human indeterminacy, when exact statistical distribution of probabilities is not-known, until the decision maker is explicitly considering them, or taking a firm decision. Multiple applications of QP calculus to various decision contexts showed that the axiomatics of QP and its mathematical apparatus can aid in modeling of human preferences that go beyond basic decision making tasks. It has been also shown in experimental studies that preferences and beliefs cannot be simply retrieved by human cognitive states, but are constructed in the decision making process, see applications to cognition, economics and finance by [7], [10], [20], [29], [21] and [42].

In this vein, quantum probability analogue of the formula (4, is based on the addition of complex probability amplitudes (and taking their square) rather than the straightforward addition of classical probabilities. This formula can be also derived for multiple realizations of the random variable A, corresponding to addition of multiple (unobserved) paths:

$$P(B=\beta) = \sum_{j} P(A=\alpha_k)P(B=\beta|A=\alpha_k) \quad (4)$$

$$+2\sum_{k<m}\cos\theta_{k,m}\sqrt{P(A=\alpha_k)P(b=\beta|a=\alpha_k)P(a=\alpha_m)P(b=\beta|a=\alpha_m)}$$

In formula, 4, an extension to multiple realizations of observables is possible, to allow for modeling of interference patterns in more complex reasoning scenarios, consisting of multiple layers of uncertainty.[c]

3 CONTEXTUALITY OF HUMAN BELIEFS

Contextuality of human beliefs, as well as contextual information processing is often associated with ambiguity in respect to the firm distribution of preferences and an incompatibility of questions a decision maker is facing. As firstly articulated in [27]- [28], it is meaningless to accommodate the decisions and expectations of decisions makers for a concrete context (state), if tomorrow the context will change.

4 APPLICATIONS TO ECONOMICS AND DECISION THEORY

QP was successfully applied to model a variety of psychological effects in economics and beyond. First applications endeavoured to provide descriptive accounts of various probabilistic fallacies, such as *order and disjunction effect*, as well as related *disjunction and conjunction fallacies.* In works of [8]- [9], interference parameters were devised to capture the incompatibility of judgements and expectations that characterize the emergence of order effect and the conjunction fallacy. There are multiple applications that address the existence of disjunction effect in economic and financial decisions, [28], [10], [20], [21] and [35].

5 RATIONALITY OF MARKET AGENTS AND THE ROOTS OF NON-CONSEQUENTIAL REASONING

In economics, rationality of market agents is formalized via a core axiom for reasoning under uncertainty, the "Savage Sure Thing Principle", (STP) by Savage in [41], p.21. In non-technical-terms it postulates:

If you prefer prospect b_+ to prospect b_- given that a possible future event A happens $(a=+1)$, and you also prefer prospect b_+ still if future event A does not happen $(a=-1)$; then you should prefer prospect b_+, despite having no knowledge of whether or not event A realized.

Following the above principle, a decision maker's probabilistic estimates and their update ought to follow a Bayesian conditioning procedure; consequently, the state space of outcomes on the agent's personal utility curve would obey the principle of additivity of the state space, i.e. with the probabilities being linearly additive and summing up to one. Also, probability is treated as CP endowed with the *subjective inter-*

[c] The indexes k and m capture the impact of the phase angles associated with the basis vectors - corresponding to events in CP.

pretation of probability.

Savage exemplified how STP works in practice, with the aid of a widely cited story of a businessman, who contemplates buying a property before the Presidential elections in the US. The businessman is uncertain whether the Republican Party, or the Democrats will win the election campaign. He decides that he will buy the property if the Republicans win, but also he decides that he should buy the property even if the Democrats win, for instance by considering that the property will appreciate given both states of the world.

STP is regarded in neo-classical economics as an axiom of rationality of decision makers that is closely connected with the core assumption of standard finance theory on "perfect foresight" of agents. Rational investors are able to evaluate states of the world and corresponding consequences (state-prices) of their investment decisions.[d]

The principle could be statistically represented with the aid of FTP. Accommodating the decision making frequencies in FTP can be used as a mathematical test of the degree of rationality for an ensemble of agents, to ascertain the strength of preference reversals, see a preliminary discussion in [44]. The roots of non-consequential reasoning that can be observed in studies on violation of STP are mainly due to a violation of the reduction axiom (that also leads to the non- satisfaction of the all-important independence axiom) and hence, an inability of a decision maker to form a joint distribution over outcomes. A core vindication in behavioral economics and finance literature, is due to agents being prone to biased behaviour and a usage short-cuts instead of than contemplating through all branches of the decision tree to form expectations, see reviews in [15] and [26]. We also highlight that the implications of non-consequential reasoning, whereby a decision maker is prone to informational frames are far-reaching for economic and financial decisions. For illustration, in an investment setting, [1] shows in a series of experiments that agents tend to hold sub-optimal portfolios, when the frequency of information about return realization is manipulated.

6 UNCERTAINTY IN FINANCIAL MARKETS

Financial markets are often characterized by asymmetric information and heightened levels of uncertainty about the dynamics of the payoff relevant variables, e.g., economic variables and companies' fundamentals. Nowadays, there are many complex financial products, from various types of derivatives to new financial instruments, such as cryptocurrencies. For these types of financial assets historical observations might not exits that further complicate the formation of expectations about the return distribution and the judgements about their fundamental values, [46].

Uncertainty was early separated from the objective risk, which entails all probabilities and state realizations being explicitly conveyed to the deci-

[d] We note that STP was formulated by Savage as part of the "Subjective Expected Utility Theory".

sion maker. In his seminal work on risk and uncertainty, Knight (1921, p.19), [36] draws on the distinction between these two concepts: *"Uncertainty must be taken radically distinct from the familiar notion of Risk...It will appear that a measurable uncertainty or "risk" proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all".* In order to be able to measure risk or uncertainty we have to assign some probabilities, which can be distinguished between *objective* (frequentionist approach) and *subjective* probabilities, based on individual estimates that follow a Bayesian updating scheme.

The renowned urn experiment by [16] that is considered as the etalon for all future studies on the mismatch between subjective and objective risks' perception, showed that a 'non-knowledge' about the exact probability distribution by the decision makers, triggers an avoidance of ambiguous outcomes. More broadly, ambiguity refers to a situation, in which the consequences and the states are known to the decision makers, yet their probabilistic composition is vaguely known, or not attainable at all. Many choices in real life decision making scenarios are characterized by outcomes and states that were never or rarely observed (e.g., financial crises, catastrophes and other types of 'extreme events').

In finance setting, ambiguous knowledge can lead to various forms of asset miss-pricings, such as ambiguity premiums on risky assets demanded by investors, cf., review of CP-based (and non-linear modifications thereof) ambiguity models in [17]. Furthermore, Halevy, [22] shows experimentally that non-classical information processing and ambiguity attitudes can be closely linked. By introducing Ellsberg urns that resemble compound lotteries with known and unknown risks he shows experimentally that aversion to ambiguity goes hand in hand with a non-satisfaction of the reduction axiom. This finding echoes the conclusions of experiments on non-consequential reasoning, in which participants avoid taking important decisions, when they do not think through the decision tree of possible state-consequences.

Quantum probability can serve as a suitable tool to capture this deeper form of non-classical uncertainty with the aid of (quantum) states of superpositions of beliefs.[e] Economic agents can also possess ambiguous priors that affect their posterior expectations, even if the upcoming information is non-noisy. Authors in [35] devise a QP based asset trading framework under ambiguous beliefs, where two ensembles of agents are uncertain about the future distribution of dividends and hence cannot attain the exact return distribution of a risky asset. One ensemble of agents is characterized by a state of optimism and is making trades based on their ambiguous expectations, tinted by ambiguity attraction. In other investment periods, pessimistic beliefs kick in, and all agents are trading based on their pessimistic beliefs, thereby yielding deflationary pressures on an asset's price. A quantum dynamical model from an

[e] Heterogeneous states can be also modeled, via the introduction of "mixed states".

approximation of Markovian dynamics is devised in [31], to provide a price realization prognosis for a multi-asset configuration. The model is inspired by the findings of behavioural finance, where resolution of agents' ambiguous beliefs gives raise to changes in assets' price configuration that can give volatility jumps and price realizations that deviate from fundamental values. An important feature of the price states behaviour is that agents can be over-optimistic about the future price growth under the a lack of complete information. Prolonged bubbles that are not supported by the dynamics of fundamental variables can sustain, see a breakdown of empirical evidence related to surfacing of housing bubbles and other financial bubbles in [46].

In this setting one can pose a question on, whether a non-satisfaction of the cannons of CP reasoning automatically implies deviation from rationality?

6.1 Some Notes on Biases and Irrational Behaviour

As noted, conveyed information plays a pivotal role in human probabilistic judgements, whereby the explicitness of an event's description may eliminate the disjunction effect, or trigger an aggregated rather than a segregated evaluation of relevant outcomes. Context is not only hidden in the questions and problems' frames, but in the whole surrounding environment of the memory and the knowledge of the decision maker. Informational 'noise' coming from all surrounding visual and verbal sources, similarly contributes to the time-dependent evolution of a person's cognitive state, cf. [48], [10], [29], and references herein. Heuristics and biases research can be considered as a helpful tool-kit to explain the cognitive processes that underpin human decision making in various contexts. Heuristics can be defined as mental short-cuts, aimed at minimizing decision making time and amount of effort, when confronted with more complex random variables - which are difficult to evaluate in probabilistic terms. Reason-based choice paradigm can also give a compelling explanation to preference reversals, where different criteria and thereby reasons for the particular preferences can appear as the state of the world changes, e.g., frameworks in [26], [43], [44]. An important assumption in these qualitative accounts is the changing nature of the decision making context. Reasons can be also connected with motivation and ethics (due to the scope of this contribution just mentioned in brief). As the decision making frame changes, the new context can trigger a different ranking of preferences. Other-regarding preference can also emerge. As an example, experimental findings in [45] show that a state of financial deprivation affects the preferences of agents that is closely related to their morality. In a game-theoretic setting, one can observe that a growth in inequality leads to less cooperation among group members, [19]. While the level of wealth of agents does not change, the knowledge about the distribution of economic resources among other agents can strongly affect the moral standards and preferences for cooperation.

We can conclude that the observed decision making paradoxes and violations of rationality axioms can be explained via the broader impact of mental factors (complex interaction of motivation, biases and probability judgements in different decision making states), as well as the impact of external frames. It should be also noted that frames are often bringing to light biases and change the underlying reasons for preferences. This pattern of decision making can be coined as "contextual", since not only the consequences, but the realized decision making state matters.

In order to avoid case by case explanations, it is useful to introduce a more general contextual description of human expectations and preferences.[f]

By using the QP calculus and its axiomatics, it is possible to capture the deviations from FTP and Bayesian update. Following the tenets of standard economic and finance models, agents that do not obey the laws of CP would be coined as being not *probabilistically sophisticated*, and as a result being irrational in their preference formation. However, one could question the CP approach to the notion of rationality, and define different type of rationality, as one respecting the quantum calculus of probabilities and the quantum formula for interference of probabilities, see a detailed review in [20] and [42].

7 QUANTUM PROBABILITY MODELS OF CONTEXTUAL PREFERENCES IN POLITICS

Politics is regarded as a vital area of social science and relies strongly on the assumption of voters' rationality, implying a stability of preferences. People would naturally follow the same principles, i.e., the axioms of rationality, in their political decisions as in other situations, such as investment decisions. The rational choice paradigm of modern decision theories was naturally conveyed into decision making domains other than economics, namely to political decisions and in particular to Voting Theory. Political decision making is a special sphere, where humans have to make decisions with far-reaching implications for themselves and the society in general. The types of decisions can involve ballot-casting in different elections, from local to governmental. Similarly, on the party level, the involved parties as political entities have the responsibility to strategically plan their political actions, taking into consideration all possible consequences. A large body of research in decision theory operated on the assumption that the domain of the utility function can also be used to derive personal utility over a range of consequences beyond monetary outcomes, [25]. Authors in [49] highlight that voters, who strive to maximize their returns in terms of, for example, support of their ideological position

[f] We note that there are many successful decision theoretic models that aim to generalize the expected utility approach and usage of the CP scheme. These models can also capture impact of some specific biases, most notably, loss aversion in respect to an initial state of wealth and expectations, known as the "reference point", [26].

or some policy, would retain a consistency of preferences. If the outcomes of elections are uncertain, voters are supposed to analyse information in a subjective manner, by following the axioms of classical probability theory and the Bayesian updating scheme.

Another feature of political decisions is related to their inter-temporal dimensions and interrelation. Consequences of choices can emerge at a later stage and the possible decisions on one issue can strongly depend on the preferences regarding other political issues. The complex nature of political decisions posed a challenge to Savage axioms. One of the most puzzling effects that has been recently explored in political choice theory is the so-called effect of *non-separability*. Lacy (2001, p.239) [38] defines this effect: *"A person has non-separable preferences when her preferences on an issue or set of issues depend on the outcome of other issues"*.

7.1 Non-separability in Political Decisions

Non-separability, or strong interrelation of (political) decisions has been explored in more recent political studies. A spatial representations of such preferences was devised, see [38] and [14]. These studies show that preferences of voters and also of governments are often not evolving in isolation: the issues and their outcomes are not unconditioned and unconstrained, but irreducibly connected with each other by the decision-making states of the subjects.[g] We briefly outline the characteristics of non-separability, as defined in political science, adopting a classical definition from [38], p. 240.

Let $\mathbf{J} = \{1, ..., J\}$ be a set of issues. Let $\mathbf{o} = (o_1, ..., o_j)$ be a $\mathbf{J}$-tuple of outcomes across all J issues. Define x and y as mutually exclusive and exhaustive non empty subsets of the $\mathbf{o}$. x' is an outcome that differs from x on at least one issue, and y' differs from y on at least one issue. Now suppose individual i has a reflexive and transitive weak preference relation[h], $\succeq_i$, ordering all $\mathbf{J}$-tuples of policy outcomes. Then $i's$ preferences are:

- separable iff for all x, y, y', $(x, y) \succeq_i (x', y)$ and $(x, y') \succeq_i (x', y')$.
- completely non-separable iff for all x there exists a y and y' such that $(x, y) \succeq_i (x', y)$ and $(x', y') \succeq_i (x, y')$.

Non-separability reveals a more complex nature of human preferences-namely an interdependence of preferences. In contrast to what is often assumed in traditional political science studies, preferences are not fixed over time and isolated from other decision-making contexts. In political literature, this phenomenon has been mainly studied among voters, due to the possibility of obtaining detailed statistics through surveys and opinion/exit polls. Non-separability of governmental and party

[g] To put it differently, the decisions of the voters affect the consequences in the different states of the world, which is at variance with Savage axioms and notably, STP.

[h] To establish a formal representation of a preference relation, economic axioms serve as building blocks that allow us to establish an ordering of preferences. Reflexivity stems from the axiom of preference completeness and pertains to a preference equivalence, where e.g., for outcomes x and y, $x \sim y$, iff $x = y$.

decisions has not been so widely explored, however [14] present statistics on the existence of EU member states' non-separable behaviour related to several political issues.

7.2 QP Models of Voters' Non-separable Behaviour in US Elections

The US political system has a governance scheme based on divided partisan control formed by the executive power attributed to the President of the U.S. and the legislative power vested in the U.S. Congress. Moreover, the political arena was historically dominated by two parties, the Democrats and the Republicans, such that the U.S. is commonly regarded as a country with an established two party political system. Historically, voters used to hold stable preferences by supporting the same political party in both the White House and (at least one) Houses of Congress elections. In this regard, a large body of orthodox studies on voting preferences perceived such power accumulation as a matter of fact and argued that Divided government should be perceived as a negative occurrence that inhibits the normal functioning of political system ([13], [39], [3]). In line with the postulates of expected utility theories, voters would naturally choose the same party in both types of elections, obeying principles of completeness and invariance. Over the last 40 years, the situation of power distribution in U.S. politics began to change and the voters started to seek to separate political power, at least based on the results of election campaigns that led to the domination of Congress and White House by different political parties. The first attention paid to this phenomenon was by a well-known political scientist, Morris Fiorina [18], who explained this phenomenon not as a random occurrence, but as a purposive (but not necessarily conscious) motivation of voters to balance political power so as to sustain a less extreme political course. Study by [47] shows that voters are highly influenced by the informational context related to the outcomes of elections. In particular, voters strongly relate the outcomes of the Presidential elections to their subsequent voting decisions, and often change their preferences in favour of another party during the Congress elections. The phenomenon of non-separability of choices implies that, on a global political scale, a particular informational context affects voters' belief states in a mode that is incompatible with their previous attitudes. In fact, according to [47], more than half of the respondents exhibited preference reversals in respect to their Presidential choices, after information on the outcome of Congress elections was presented to them. This effect was amplified for the undecided voters who did not know, whom they would prefer as a President in the baseline questions. In a sense, additional information did elicit the voting preferences of this cohort of respondents. In line with psychological studies in decision theory as well as investment behaviour, opinion poll studies unearth a deep contextuality of individual preferences in a political environment.

Authors in [30] and [34] performed a theoretical analysis of the statistical data from opinion poll studies by embedding the voting frequencies into FTP. It was shown showed that FTP is not satisfied and order effects exist in the voting for Presidency and Congress. The disjunctions of conditional probabilities show a sub-additivity in respect to the marginal probability, which indicates a form of reverse disjunction effect (i.e. uncertainty attraction in respect to voting for a Democratic candidate, when no information about Congress is given). A quantum formula of total probability was able to quantify the contextuality of voters' preferences via the size of the interference term and computation of the phase angle.[i] We can conclude that the size of interference effects can provide a measure of preference reversals in different decision making contexts, beyond monetary payoff outcomes. In works by [4] and [7] the authors seek to consider how different interference terms can serve as a predictor of agents subjective evaluation of risk in a setting of risky lottery selection.

7.3 Non-Separability and Entanglement

Authors in [49] conjectured that non-separability of preferences that is often detected in voting theory and more broadly in politics, can have its roots in a "quantum type" of non-separability that captures more strong correlations between observables and their realizations. Works by [33] and [34] incorporate a time-dependent state dynamics, to accommodate the impact of external information (an informational "bath") upon decision formation of voters. Similarly, contributions by [6] and [32] applied quantum dynamical models to model alliance and coalition formation of political parties in different political systems. The latter work shows that coalition formation is a complex process, where similar preference states of both parties are key to their agreement. These preferences are depicted through entangled states of both parties that are necessary to achieve to a steady state of their mutual political cooperation.

Remark on operational modeling of entangled states:

If we look at a tensor product $H = H_1 \otimes H_2 \otimes ... \otimes H_n$ of Hilbert spaces $H_k, k = 1, 2, ..., n$ we can note that these states of the total Hilbert space H can be either *separable* or *non-separable*. Non-separable states that cannot be considered in isolation are called entangled states. We start by considering some pure states and their representation as separable and non-separable states. The states from the first class, i.e., separable pure states, can be represented in the form:

$$|\psi\rangle = \psi_1 \otimes \cdots \otimes \psi_n \equiv |\psi_1 ... \psi_n\rangle, \quad (5)$$

where $|\psi_i\rangle \in H_i$. States, which cannot be represented in this way are called non-separable, entangled. Essentially, the mathematical representation of entanglement is straightforward, and means an impossibility of tensor product factorization (for more details we refer to readings by [23] and [40].).

[i] In [34] a quantum type of constructive interference was observed. We can contrast it with the destructive interference pattern observed in inter-temporal investment game experiment by [21].

8 DOES VIOLATION OF CP RULES IMPLY "QUANTUM- TYPE" OF CONTEXTUALITY?

In principle, there is no reason to expect that all statistical data on beliefs and preferences would respect the constraints of QP. The most famous QP constraint is *Tsirelson's bound* that loosely speaking is a measure of an upper limit for quantum mechanical correlations. We recall that Bell's type inequalities are given as special linear combinations of pairwise correlations of observables. Such inequalities are satisfied under the assumption that these correlations are described via CP and a joint probability distribution over the outcomes exists. Violation of Bell's type inequalities would imply that QP rather than CP is a suitable model to accommodate these pairs of correlated observables. The most famous of these Bell's type inequalities is 'CHSH-inequality', due to John Clauser, Michael Horne, Abner Shimony, and Richard Holt, [12].

If we consider two pairs of observables, A_1, A_2 and B_1, B_2 and their cross-pair correlations, e.g., A_1 and B_1 can be measured jointly, but the observables inside a pair (A_1, A_2) would be *incompatible, i.e., they cannot be measured jointly.* If we denote the CHSH linear combination of correlations as C_{CHSH}, then for a CP-model the upper limit would be :

$$|C_{\text{CHSH}}| \leq 2.$$

For a general non-classical model:

$$|C_{\text{CHSH}}| \leq 4.$$

Whereby for a QP-model, we have a specific bound:

$$|C_{\text{CHSH}}| \leq 2\sqrt{2}.$$

Can we say that correlations for observations on cognitive statistics are obeying the constraints required for QP model?

In cognitive experiments, A_1, A_2 and B_1, B_2 are treated as two pairs of questions which are asked (in total 4 questions). Cross-pair questions can be asked *jointly*, but the questions inside a pair cannot. In principle, there is no reason to expect that outcomes (answers) on question-observables would obey the QP- bound. Testing of Bell's type inequalities for cognitive variables in not a straightforward task, due to signalling problems (direct impact of the informational content of the questions). For instance, in pioneering experiments by [5,11] the CP bound (=2) was not respected, indicating that the psychological data cannot be accommodated in a CP model, unless some hidden variables are considered.

8.1 Summarizing Remarks

We can see that the above mentioned instances of "contextuality" that account for irrational decision making are strongly shaped by the external decision making context, such as complexity and uncertainty, arrival of irrelevant information, order effects and other 'noisy' factors. In addition to the external environment, the internal mental conditions of a decision maker play a role. Equal attitude towards risk and uncertainty, as well as classical probabilistic assessment of information and formation of equal prior beliefs among decision makers appears not to hold. We reviewed theoretical and experimental studies that aimed to quantify the deviations of human expectations and decisions from the cannons of CP and rational preference formation. At the same time, a more general probability theory, QP calculus can serve as a unified mathematical framework that can capture in a systematic way the "biased" and "irrational" preferences of people in various decision scenarios. Further experimental tests on human contextual preferences are crucial, to testify its predictive and potentially normative status.

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